

Examples of Different Results using car package

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1 Tested Version and Books used for the Validation

1.1 Packages Used

- 'sasLM' version: 0.10.8
- 'SAS' version: 9.4 Licensed and University Edition
- 'car' version: 3.1.5
- R version: R version 4.5.2 (2025-10-31 ucrt)

The 'car' package is not necessary for 'sasLM.' It is used for the comparison of the results.

If you see any difference between 'car' and 'sasLM', 'SAS' results coincide with 'sasLM', not with 'car.'

Before 'sasLM' is available on CRAN, you can download using the following command in R.

```
install.packages("sasLM", repos="http://r.acr.kr")
```

1.2 Books and Articles used for the Test

1. Snee RD. Computation and Use of Expected Mean Squares in Analysis of Variance. J Qual Tech. 1974;6(3);128-137.
2. Goodnight JH. The General Linear Models Procedure, Proceedings of the First International SAS User's Group, SAS Institute, Raleigh, N.C. 1976.
3. Littell RC, Stroup WW, Freund RJ. SAS for Linear Models 4e. John Wiley & Sons Inc. 2002.
4. Sahai H, Ojeda MM. Analysis of Variance for Random Models Volume 2 Unbalanced Data. 2005.
5. Federer WT, King F. Variations on Split Plot and Split Block Experiment Designs. John Wiley & Sons Inc. 2007.
6. Hinkelmann K, Kempthorne O. Design and Analysis of Experiments Volume 1 Introduction to Experimental Design. 2e. John Wiley & Sons Inc. 2008.
7. Searle SR, Gruber MHJ. Linear Models 2e, Kindle Edition. John Wiley & Sons Inc. 2016.

2 Snee EMS ANOVA 1974

Reference

- Snee RD. Computation and Use of Expected Mean Squares in Analysis of Variance. J Qual Tech. 1974;6(3);128-137.

(1) MODEL

```
Snee = read.csv("http://r.acr.kr/Snee_EMS_ANOVA1974.csv")
Snee = af(Snee, c("Machine", "Analyst", "Test", "Day"))
aov3(Y ~ Day/Machine/Analyst/Test, Snee)
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	167	751.27	4.4986		
Day	41	359.44	8.7669		
Day:Machine	42	199.40	4.7477		
Day:Machine:Analyst	42	118.80	2.8285		
Day:Machine:Analyst:Test	42	70.30	1.6739		
RESIDUALS	0	0.00			
CORRECTED TOTAL	167	751.27			

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(Y ~ Day/Machine/Analyst/Test, Snee), type=3, singular.ok=TRUE)
# NOT WORKING
```

3 Goodnight

Reference

- Goodnight JH. The General Linear Models Procedure, Proceedings of the First International SAS User's Group, SAS Institute, Raleigh, N.C. 1976.

3.1 p33

(2) MODEL

```
p33 = read.csv("http://r.acr.kr/Goodnight-p33.csv")
p33 = af(p33, c("A", "B"))
aov3(y ~ A + B + A:B, p33) # p35
```

Response : y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	4	34.905	8.7261		
A	1	3.028	3.0276		
B	1	23.523	23.5225		
A:B	1	0.008	0.0081		
RESIDUALS	0	0.000			
CORRECTED TOTAL	4	34.905			

```
options(contrasts = c("contr.sum", "contr.poly"))
Anova(lm(y ~ A + B + A:B, p33), type=3, singular.ok=TRUE) # NOT WORKING
```

4 SAS for Linear Models 4e

Reference

- Littell RC, Stroup WW, Freund RJ. SAS for Linear Models 4e. John Wiley & Sons Inc. 2002.

4.1 p403

(3) MODEL

```
p403 = read.table("http://r.acr.kr/sas4lm/p403.txt", header=TRUE)
p403 = af(p403, c("PATIENT", "VISIT"))
aov3(HR ~ SEQUENCE + PATIENT %in% SEQUENCE + VISIT + DRUG + RESIDS + RESIDT, p403)
```

Response : HR

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
MODEL	29	6408.7	220.989	3.9120	3.127e-05	***
SEQUENCE	5	701.2	140.237	2.4825	0.04665	*
VISIT	2	146.8	73.389	1.2991	0.28350	
DRUG	2	344.0	171.975	3.0443	0.05826	.
RESIDS	1	309.2	309.174	5.4731	0.02414	*
RESIDT	1	0.8	0.840	0.0149	0.90351	
SEQUENCE:PATIENT	18	4692.3	260.685	4.6147	2.210e-05	***
RESIDUALS	42	2372.6	56.490			
CORRECTED TOTAL	71	8781.3				

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(HR ~ SEQUENCE + PATIENT %in% SEQUENCE + VISIT + DRUG + RESIDS + RESIDT,
p403), type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: HR

	Sum Sq	Df	F values	Pr(>F)
SEQUENCE	0.0	0		
VISIT	146.8	2	1.2991	0.28350
DRUG	343.9	2	3.0443	0.05826
RESIDS	309.2	1	5.4731	0.02414
RESIDT	0.8	1	0.0149	0.90351
SEQUENCE:PATIENT	4692.3	18	4.6147	2.21e-05
Residuals	2372.6	42		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

4.2 p417

(4) MODEL

```
p417 = read.table("http://r.acr.kr/sas4lm/p417.txt", header=TRUE)
p417 = af(p417, c("TRT", "POT", "PLANT"))
aov3(Y ~ TRT + POT %in% TRT, p417) # p418 Output 11.28
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	7	267.226	38.175	12.433	7.522e-05 ***
TRT	2	200.111	100.055	32.586	8.626e-06 ***
TRT:POT	5	30.306	6.061	1.974	0.1499
RESIDUALS	13	39.917	3.071		
CORRECTED TOTAL	20	307.143			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(Y ~ TRT + POT %in% TRT, p417), type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: Y

	Sum Sq	Df	F values	Pr(>F)
TRT	22.310	1	7.266	0.01835 *
TRT:POT	30.306	5	1.974	0.14991
Residuals	39.917	13		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

4.3 p431

(5) MODEL

```
p431 = read.table("http://r.acr.kr/sas4lm/p431.txt", header=TRUE)
p431 = af(p431, c("line", "sire", "agedam", "steerno"))
aov3(avdlygn ~ line + line:sire + agedam + line:agedam + age + intlwt, p431)
```

Response : avdlygn

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	16	2.5275	0.15797	3.1437	0.001091 **
line	2	0.1362	0.06810	1.3553	0.267560
agedam	2	0.1301	0.06505	1.2946	0.283392
age	1	0.3813	0.38128	7.5878	0.008277 **
intlwt	1	0.2697	0.26970	5.3674	0.024830 *
line:sire	6	0.9739	0.16231	3.2303	0.009543 **
line:agedam	4	0.4534	0.11336	2.2560	0.076821 .
RESIDUALS	48	2.4119	0.05025		
CORRECTED TOTAL	64	4.9394			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

p433 Output 11.40

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(avdlygn ~ line + line:sire + agedam + line:agedam + age + intlwt, p431),
      type=3, singular.ok=TRUE) # NOT OK for line
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: avdlygn

	Sum Sq	Df	F values	Pr(>F)
line	0.00000	0		
agedam	0.13011	2	1.2946	0.283392
age	0.38128	1	7.5878	0.008277 **
intlwt	0.26970	1	5.3674	0.024830 *
line:sire	0.97389	6	3.2303	0.009543 **
line:agedam	0.45343	4	2.2560	0.076821 .
Residuals	2.41192	48		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

5 Sahai - Unbalanced

Reference

- Sahai H, Ojeda MM. Analysis of Variance for Random Models Volume 2 Unbalanced Data. 2005.

5.1 Table 15.3

(6) MODEL

```
T15.3 = read.table("http://r.acr.kr/sahai/T15.3.txt")
colnames(T15.3) = c("Dam", "Sire", "pH")
T15.3 = af(T15.3, c("Dam", "Sire"))
aov3(pH ~ Dam/Sire, T15.3) # p301
```

Response : pH

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	36	0.25804	0.0071678	2.8977	7.200e-06 ***
Dam	14	0.17940	0.0128146	5.1805	1.347e-07 ***
Dam:Sire	22	0.08002	0.0036374	1.4705	0.09662 .
RESIDUALS	123	0.30425	0.0024736		
CORRECTED TOTAL	159	0.56229			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts = c("contr.sum", "contr.poly"))
Anova(lm(pH ~ Dam/Sire, T15.3), type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: pH

	Sum Sq	Df	F values	Pr(>F)
Dam	0.081011	6	5.4584	4.898e-05 ***
Dam:Sire	0.080024	22	1.4705	0.09662 .
Residuals	0.304253	123		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

5.2 Table 16.3

(7) MODEL

```
T16.3 = read.csv("http://r.acr.kr/sahai/T16.3.csv")
colnames(T16.3) = c("Plot", "Sample", "Subsample", "Residue")
T16.3 = af(T16.3, c("Plot", "Sample", "Subsample"))
aov3(Residue ~ Plot/Sample/Subsample, T16.3) # p344
```

Response : Residue

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	54	3.1897	0.059069	5.8842	1.476e-05 ***
Plot	10	1.7869	0.178686	17.7998	2.547e-08 ***
Plot:Sample	22	0.9917	0.045079	4.4906	0.0004209 ***
Plot:Sample:Subsample	22	0.3576	0.016253	1.6191	0.1330632
RESIDUALS	22	0.2208	0.010039		
CORRECTED TOTAL	76	3.4106			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts = c("contr.sum", "contr.poly"))
Anova(lm(Residue ~ Plot/Sample/Subsample, T16.3), type=3, singular.ok=TRUE)
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: Residue

	Sum Sq	Df	F values	Pr(>F)
Plot	0.00000	0		
Plot:Sample	0.36613	11	3.3156	0.00805 **
Plot:Sample:Subsample	0.35758	22	1.6191	0.13306
Residuals	0.22085	22		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

NOT OK

6 Federer - Variations

Reference

- Federer WT, King F. Variations on Split Plot and Split Block Experiment Designs. John Wiley & Sons Inc. 2007.

6.1 Example 2.2

(8) MODEL

```
ex2.2 = read.table("http://r.acr.kr/split/sbex2_2.txt", header=TRUE)
ex2.2 = af(ex2.2, c("Row", "Column", "R", "S"))
aov3(Y ~ Row + R + S + R:S + Row:R + Column:S + Column:R:S, ex2.2)
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	99	22310.4	225.36		
Row	0				
R	4	1159.8	289.94		
S	3	351.9	117.29		
R:S	12	826.0	68.83		
Row:R	0				
S:Column	12	3863.3	321.94		
R:S:Column	48	11982.3	249.63		
RESIDUALS	0	0.0			
CORRECTED TOTAL	99	22310.4			

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(Y ~ Row + R + S + R:S + Row:R + Column:S + Column:R:S, ex2.2), type=3,
      singular.ok=TRUE) # NOT WORKING
```

6.2 Example 3.1

(9) MODEL

```
ex3.1a = read.table("http://r.acr.kr/split/Ex3.1-example.txt", header=TRUE)
ex3.1a = af(ex3.1a, c("row", "P", "column", "R", "S"))
aov3(height ~ row + R + P + S + S:R + row:P + R:P + row:R:P + S:P + S:P:row +
      S:R:P + R:S:P:row, ex3.1a)
```

Response : height

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	199	7534.8	37.86		
row	4	2017.0	504.26		
R	4	90.6	22.66		
P	1	253.1	253.13		
S	3	16.4	5.46		
R:S	12	195.0	16.25		
row:P	4	167.2	41.81		
R:P	4	505.0	126.24		
P:S	3	14.3	4.77		
row:R:P	32	2933.5	91.67		
row:P:S	24	234.7	9.78		
R:P:S	12	100.3	8.36		
row:R:P:S	96	1007.5	10.49		
RESIDUALS	0	0.0			
CORRECTED TOTAL	199	7534.8			

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(height ~ row + R + P + S + S:R + row:P + R:P + row:R:P + S:P +
      S:P:row + S:R:P + R:S:P:row, ex3.1a), type=3, singular.ok=TRUE)
# NOT WORKING
```

6.3 Appendix 3.1 p94

(10) MODEL

```
ex3.1b = read.table("http://r.acr.kr/split/spexvar3.txt", header=TRUE)
ex3.1b = af(ex3.1b, c("rep", "var", "nit", "row", "col"))
aov3(yield ~ rep + var + rep:var + nit + var:nit + row + col, ex3.1b)
```

Response : yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	37	48090	1299.7	11.3414	6.734e-11 ***
rep	2	5943	2971.3	25.9273	1.449e-07 ***
var	2	2800	1399.9	12.2155	0.0001005 ***
nit	3	11978	3992.6	34.8397	1.775e-10 ***
row	9	945	105.0	0.9162	0.5230151
col	2	3171	1585.7	13.8373	4.012e-05 ***
rep:var	4	998	249.4	2.1767	0.0926008 .
var:nit	6	478	79.6	0.6949	0.6553307
RESIDUALS	34	3896	114.6		
CORRECTED TOTAL	71	51986			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(yield ~ rep + var + rep:var + nit + var:nit + row + col, ex3.1b),
      type=3, singular.ok=TRUE) # NOT OK for var
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: yield

	Sum Sq	Df	F values	Pr(>F)
rep	5942.5	2	25.9273	1.449e-07 ***
var	0.0	0		
nit	11977.9	3	34.8397	1.775e-10 ***
row	945.0	9	0.9162	0.5230
col	3171.5	2	13.8373	4.012e-05 ***
rep:var	997.8	4	2.1767	0.0926 .
var:nit	477.8	6	0.6949	0.6553
Residuals	3896.4	34		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

6.4 Example 5.1

(11) MODEL

```
ex5.1 = read.table("http://r.acr.kr/split/sbsp.txt", header=TRUE)
ex5.1 = af(ex5.1, c("R", "A", "C", "B", "Tx"))
aov3(Y ~ R + A + A:R + C + B + B:C + Tx + A:Tx + B:Tx, ex5.1)
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
MODEL	24	196.238	8.1766	7.0476	0.0008758	***
R	2	22.186	11.0928	9.5611	0.0039244	**
A	1	15.185	15.1853	13.0886	0.0040418	**
C	2	1.010	0.5049	0.4352	0.6578395	
B	1	1.792	1.7922	1.5448	0.2397515	
Tx	5	103.333	20.6667	17.8131	6.055e-05	***
R:A	2	27.426	13.7132	11.8197	0.0018198	**
C:B	2	0.085	0.0424	0.0366	0.9642020	
A:Tx	4	2.655	0.6636	0.5720	0.6886524	
B:Tx	4	2.050	0.5126	0.4418	0.7761730	
RESIDUALS	11	12.762	1.1602			
CORRECTED TOTAL	35	209.000				

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(Y ~ R + A + A:R + C + B + B:C + Tx + A:Tx + B:Tx, ex5.1),
      type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: Y

	Sum Sq	Df	F values	Pr(>F)	
R	22.186	2	9.5611	0.003924	**
A	0.000	0			
C	1.010	2	0.4352	0.657839	
B	0.000	0			
Tx	103.333	5	17.8131	6.055e-05	***
R:A	27.426	2	11.8197	0.001820	**
C:B	0.085	2	0.0366	0.964202	
A:Tx	2.655	4	0.5720	0.688652	
B:Tx	2.050	4	0.4418	0.776173	
Residuals	12.762	11			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(12) MODEL

```
aov3(Y ~ R + A + A:R + C + B + C:B + Tx + A:Tx + B:Tx + A:B:Tx, ex5.1)
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
MODEL	28	204.200	7.2929	10.6354	0.0017194	**
R	2	28.112	14.0562	20.4986	0.0011846	**
A	1	14.655	14.6551	21.3720	0.0024176	**
C	2	0.471	0.2356	0.3436	0.7205632	
B	1	1.769	1.7694	2.5804	0.1522328	
Tx	5	103.815	20.7630	30.2793	0.0001336	***
R:A	1	2.017	2.0174	2.9420	0.1300172	
C:B	1	0.644	0.6445	0.9399	0.3646045	
A:Tx	4	2.951	0.7378	1.0760	0.4358837	
B:Tx	4	3.553	0.8882	1.2954	0.3579988	
A:B:Tx	4	7.962	1.9905	2.9029	0.1038803	
RESIDUALS	7	4.800	0.6857			
CORRECTED TOTAL	35	209.000				

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(Y ~ R + A + A:R + C + B + C:B + Tx + A:Tx + B:Tx + A:B:Tx, ex5.1),
      type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: Y

	Sum Sq	Df	F values	Pr(>F)	
R	11.643	1	16.9793	0.004456	**
A	0.000	0			
C	0.002	1	0.0025	0.961483	
B	0.000	0			
Tx	89.178	3	43.3503	6.87e-05	***
R:A	2.017	1	2.9420	0.130017	
C:B	0.644	1	0.9399	0.364604	
A:Tx	0.543	3	0.2640	0.849381	
B:Tx	3.384	3	1.6451	0.264128	
A:B:Tx	7.962	4	2.9029	0.103880	
Residuals	4.800	7			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

6.5 Example 7.1

(13) MODEL

```
ex7.1 = read.table("http://r.acr.kr/split/asped.txt", header=TRUE)
ex7.1 = af(ex7.1, c("R", "G", "F"))
aov3(Y ~ R + G + R:G + F + F:G, ex7.1)
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
MODEL	95	577.83	6.0824	5.3082	1.068e-05	***
R	3	5.75	1.9167	1.6727	0.1994	
G	27	343.48	12.7216	11.1025	4.286e-08	***
F	2	50.50	25.2525	22.0385	3.686e-06	***
R:G	9	11.75	1.3056	1.1394	0.3749	
G:F	54	77.98	1.4441	1.2603	0.2718	
RESIDUALS	24	27.50	1.1458			
CORRECTED TOTAL	119	605.33				

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
```

```
Anova(lm(Y ~ R + G + R:G + F + F:G, ex7.1), type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: Y

	Sum Sq	Df	F values	Pr(>F)
R	0.000	0		
G	202.417	3	58.8848	3.258e-11 ***
F	50.505	2	22.0385	3.686e-06 ***
R:G	11.750	9	1.1394	0.3749
G:F	77.983	54	1.2603	0.2718
Residuals	27.500	24		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

6.6 Example 7.3

(14) MODEL

```
ex7.3 = read.table("http://r.acr.kr/split/assped.txt", header=TRUE)
ex7.3 = af(ex7.3, c("R", "T", "G", "F"))
aov3(Y ~ R + T + R:T + G + G:T + R:T:G + F + F:T + F:G + F:G:T, ex7.3)
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
MODEL	155	656.12	4.233	13.4461	3.997e-14	***
R	3	12.49	4.162	13.2206	5.655e-06	***
T	1	11.16	11.158	35.4430	8.021e-07	***
G	22	389.01	17.682	56.1668	< 2.2e-16	***
F	2	120.56	60.282	191.4828	< 2.2e-16	***
R:T	3	1.15	0.384	1.2206	0.316281	
T:G	22	18.42	0.837	2.6601	0.004445	**
T:F	2	0.82	0.411	1.3060	0.283432	
G:F	44	23.47	0.533	1.6943	0.053191	.
R:T:G	12	8.78	0.731	2.3235	0.025315	*
T:G:F	44	10.74	0.244	0.7753	0.790640	
RESIDUALS	36	11.33	0.315			
CORRECTED TOTAL	191	667.45				

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(Y ~ R + T + R:T + G + G:T + R:T:G + F + F:T + F:G + F:G:T, ex7.3),
      type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients
 sums of squares computed by model comparison

Anova Table (Type III tests)

Response: Y

	Sum Sq	Df	F values	Pr(>F)
R	0.000	0		
T	0.000	0		
G	73.444	2	116.6471	< 2.2e-16 ***
F	120.563	2	191.4828	< 2.2e-16 ***
R:T	0.000	0		
T:G	5.778	2	9.1765	0.0006018 ***
T:F	0.822	2	1.3060	0.2834316
G:F	23.469	44	1.6943	0.0531910 .
R:T:G	8.778	12	2.3235	0.0253153 *
T:G:F	10.740	44	0.7753	0.7906401
Residuals	11.333	36		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

6.7 Example 8.1

(15) MODEL

```
ex8.1 = read.table("http://r.acr.kr/split/asbed.txt", header=TRUE)
ex8.1 = af(ex8.1, c("R", "A", "B"))
aov3(Y ~ R + A + R:A + B + B:R + A:B + A:B:R, ex8.1)
```

Response : Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	104	3951.8	37.999		
R	2	372.2	186.111		
A	12	572.3	47.692		
B	8	185.8	23.231		
R:A	6	50.0	8.333		
R:B	4	87.4	21.861		
A:B	60	1012.3	16.871		
R:A:B	12	49.0	4.083		
RESIDUALS	0	0.0			
CORRECTED TOTAL	104	3951.8			

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(Y ~ R + A + R:A + B + B:R + A:B + A:B:R, ex8.1), type="III",
      singular.ok=TRUE) # NOT WORKING
```

6.8 Example 9.2

(16) MODEL

```
ex9.2 = read.table("http://r.acr.kr/split/Ex9.2-sbex.txt", header=TRUE)
ex9.2 = af(ex9.2, c("rep", "hyb", "gen"))
aov3(yield ~ rep + hyb + rep:hyb + gen + gen:rep + gen:hyb, ex9.2)
```

Response : yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	40	247.813	6.1953	4.4606	0.0011186 **
rep	1	0.167	0.1667	0.1200	0.7335481
hyb	9	66.796	7.4218	5.3437	0.0018370 **
gen	2	30.671	15.3356	11.0416	0.0009707 ***
rep:hyb	8	67.000	8.3750	6.0300	0.0011569 **
rep:gen	2	12.111	6.0556	4.3600	0.0308015 *
hyb:gen	18	60.504	3.3613	2.4201	0.0408545 *
RESIDUALS	16	22.222	1.3889		
CORRECTED TOTAL	56	270.035			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(yield ~ rep + hyb + rep:hyb + gen + gen:rep + gen:hyb, ex9.2), type=3,
      singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: yield

	Sum Sq	Df	F values	Pr(>F)
rep	0.000	0		
hyb	66.704	8	6.0033	0.0011847 **
gen	30.671	2	11.0416	0.0009707 ***
rep:hyb	67.000	8	6.0300	0.0011569 **
rep:gen	12.111	2	4.3600	0.0308015 *
hyb:gen	60.504	18	2.4201	0.0408545 *
Residuals	22.222	16		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

6.9 Example 10.1

(17) MODEL

```
ex10.1 = read.table("http://r.acr.kr/split/Ex10.1-New.txt", header=TRUE)
ex10.1 = af(ex10.1, c("Site", "Block", "A", "B", "C"))
f10.1 = Yield ~ Site/Block + A/Site + B/Site + A:B + A:B:Site + A:B:Site:Block +
      C + A:C + B:C + A:B:C + C:Site + A:C:Site + B:C:Site + A:B:C:Site
aov3(f10.1, ex10.1)
```

Response : Yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	239	1639561484	6860090	2.1620e+03	< 2e-16 ***
Site	3	552717	184239	5.8064e+01	< 2e-16 ***
A	4	1387680917	346920229	1.0933e+05	< 2e-16 ***
B	1	100939695	100939695	3.1812e+04	< 2e-16 ***
C	3	19356264	6452088	2.0334e+03	< 2e-16 ***
Site:Block	8	7062320	882790	2.7822e+02	< 2e-16 ***
Site:A	12	34068	2839	8.9470e-01	0.55301
Site:B	3	1618	539	1.6990e-01	0.91662
A:B	4	31444008	7861002	2.4775e+03	< 2e-16 ***
A:C	12	26075792	2172983	6.8483e+02	< 2e-16 ***
B:C	3	23901387	7967129	2.5109e+03	< 2e-16 ***
Site:C	9	47625	5292	1.6677e+00	0.09747 .
Site:A:B	12	33737	2811	8.8600e-01	0.56185
A:B:C	12	41996729	3499727	1.1030e+03	< 2e-16 ***
Site:A:C	36	104110	2892	9.1140e-01	0.61768
Site:B:C	9	61111	6790	2.1400e+00	0.02701 *
Site:Block:A:B	72	186911	2596	8.1810e-01	0.84155
Site:A:B:C	36	82475	2291	7.2200e-01	0.87941
RESIDUALS	240	761522	3173		
CORRECTED TOTAL	479	1640323006			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(f10.1, ex10.1), type=3, singular.ok=TRUE) # NOT OK for Site:Block
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: Yield

	Sum Sq	Df	F values	Pr(>F)
Site	552717	3	5.8064e+01	< 2e-16 ***
A	1387680917	4	1.0933e+05	< 2e-16 ***
B	100939695	1	3.1812e+04	< 2e-16 ***
C	19356264	3	2.0334e+03	< 2e-16 ***
Site:Block	0	0		

Site:A	34068	12	8.9470e-01	0.55301	
Site:B	1618	3	1.6990e-01	0.91662	
A:B	31444008	4	2.4775e+03	< 2e-16	***
A:C	26075792	12	6.8483e+02	< 2e-16	***
B:C	23901388	3	2.5109e+03	< 2e-16	***
Site:C	47625	9	1.6677e+00	0.09747	.
Site:A:B	33737	12	8.8600e-01	0.56185	
A:B:C	41996729	12	1.1030e+03	< 2e-16	***
Site:A:C	104110	36	9.1140e-01	0.61768	
Site:B:C	61111	9	2.1400e+00	0.02701	*
Site:Block:A:B	186911	72	8.1810e-01	0.84155	
Site:A:B:C	82475	36	7.2200e-01	0.87941	
Residuals	761522	240			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

7 Hinkelmann & Kempthorne - Volume 1

Reference

- Hinkelmann K, Kempthorne O. Design and Analysis of Experiments Volume 1 Introduction to Experimental Design. 2e. John Wiley & Sons Inc. 2008.

7.1 p410

(18) MODEL

```
v1p410 = read.table("http://r.acr.kr/kemp/v1p410.txt", head=TRUE)
v1p410$carry = ifelse(v1p410$carry == 0, 3, v1p410$carry)
v1p410 = af(v1p410, c("period", "sequence", "steer", "trt", "carry"))
aov3(y ~ period + sequence + steer:sequence + trt + carry, v1p410) # OK
```

Response : y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
MODEL	17	1302.51	76.618	8.7402	1.572e-05	***
period	2	172.31	86.154	9.8279	0.0013030	**
sequence	5	318.69	63.738	7.2709	0.0006954	***
trt	2	440.61	220.304	25.1311	6.164e-06	***
carry	2	16.43	8.215	0.9372	0.4100385	
sequence:steer	6	118.50	19.750	2.2530	0.0849122	.
RESIDUALS	18	157.79	8.766			
CORRECTED TOTAL	35	1460.31				

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(y ~ period + sequence + steer:sequence + trt + carry, v1p410), type=3,
singular.ok=TRUE) # NOT OK for sequence
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: y

	Sum Sq	Df	F values	Pr(>F)	
period	172.31	2	9.8279	0.001303	**
sequence	0.00	0			
trt	440.61	2	25.1311	6.164e-06	***
carry	16.43	2	0.9372	0.410038	
sequence:steer	118.50	6	2.2530	0.084912	.
Residuals	157.79	18			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

8 Searle - Linear Models 2e

Reference

- Searle SR, Gruber MHJ. Linear Models 2e, Kindle Edition. John Wiley & Sons Inc. 2016.

8.1 7.2 (p390, 59%)

(19) MODEL

```
weight = c(8,13,9,12,7,11,6,12,12,14,9,7,14,16,10,14,11,13)
treatment = c("ta","ta","ta","ta","ta","ta","tb","tb","tb","tb","tc","tc","tc",
              "tc","tc","tc","tc","tc")
variety = c("va","va","va","vc","vd","vd","va","va","vb","vb","vb","vb","vc",
            "vc","vd","vd","vd","vd")
d1 = data.frame(weight, treatment, variety)
aov3(weight ~ treatment*variety, d1)
```

Response : weight

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	7	82.000	11.7143	2.0918	0.13995
treatment	2	12.471	6.2353	1.1134	0.36595
variety	3	34.872	11.6240	2.0757	0.16719
treatment:variety	2	34.714	17.3571	3.0995	0.08965 .
RESIDUALS	10	56.000	5.6000		
CORRECTED TOTAL	17	138.000			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts = c("contr.sum", "contr.poly"))
Anova(lm(weight ~ treatment*variety, d1), type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: weight

	Sum Sq	Df	F values	Pr(>F)
treatment	0.000	0		
variety	0.000	0		
treatment:variety	34.714	2	3.0995	0.08965 .
Residuals	56.000	10		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

8.2 7.2 (p393, 60%)

(20) MODEL

```
percent = c(31,33,44,36,38,26,37,59,42,42,34,42,28,39,36,32,38,42,36,22,42,46,
            26,37,43)
refinery = c(rep("g",9),rep("n",8),rep("s",8))
process = as.factor(c(1,1,1,1,1,1,2,2,2,1,1,1,1,2,2,2,2,1,1,1,2,2,2,2,2))
source0 = c("t","t","t","t","o","m","t","t","o","m","i","i","i","t","o","m","m",
            "t","o","i","o","o","m","i","i")
d2 = data.frame(percent, refinery, process, source=source0)
aov3(percent ~ refinery*source, d2)
```

Response : percent

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	10	442.56	44.256	0.6361	0.7616
refinery	2	10.77	5.383	0.0774	0.9259
source	3	282.63	94.211	1.3542	0.2972
refinery:source	5	155.47	31.095	0.4469	0.8086
RESIDUALS	14	974.00	69.571		
CORRECTED TOTAL	24	1416.56			

```
options(contrasts=c("contr.sum", "contr.poly"))
Anova(lm(percent ~ refinery*source, d2), type=3, singular.ok=TRUE) # NOT OK
```

Note: model has aliased coefficients

sums of squares computed by model comparison

Anova Table (Type III tests)

Response: percent

	Sum Sq	Df	F values	Pr(>F)
refinery	2.52	1	0.0362	0.8518
source	268.19	2	1.9275	0.1822
refinery:source	155.47	5	0.4469	0.8086
Residuals	974.00	14		

9 Web site examples

9.1 <https://github.com/djnavarro/psyr>

(21) MODEL

```
d21 = read.csv("http://r.acr.kr/psyr/coffee.csv")
GLM(babble ~ sugar*milk - 1, d21)
```

\$ANOVA

Response : babble

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	6	472.54	78.756	298.84	2.39e-12 ***
RESIDUALS	12	3.16	0.264		
UNCORRECTED TOTAL	18	475.70			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

\$Fitness

Root MSE	babble	Mean Coef	Var	R-square	Adj R-sq
0.5133631	5.066667	10.13217	0.9933519	0.9900279	

\$`Type I`

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
sugar	3	465.64	155.213	588.9486	2.756e-13 ***
milk	1	0.96	0.956	3.6279	0.081061 .
sugar:milk	2	5.94	2.972	11.2769	0.001754 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

\$`Type II`

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
sugar	2	3.0696	1.53482	5.8238	0.017075 *
milk	1	0.9561	0.95611	3.6279	0.081061 .
sugar:milk	2	5.9439	2.97193	11.2769	0.001754 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

\$`Type III`

CAUTION: Singularity Exists !

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
sugar	2	2.1318	1.0659	4.0446	0.045426 *
milk	1	1.0041	1.0041	3.8102	0.074672 .
sugar:milk	2	5.9439	2.9719	11.2769	0.001754 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
r21 = lm(babble ~ sugar*milk - 1, d21)
```

```
Anova(r21, type=2) # NOT OK
```

Anova Table (Type II tests)

Response: babble

	Sum Sq	Df	F value	Pr(>F)	
sugar	453.76	3	573.9233	3.214e-13	***
milk	0.96	1	3.6279	0.081061	.
sugar:milk	5.94	2	11.2769	0.001754	**
Residuals	3.16	12			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
Anova(r21, type=3) # NOT OK
```

Anova Table (Type III tests)

Response: babble

	Sum Sq	Df	F value	Pr(>F)	
sugar	454.77	3	575.1970	3.172e-13	***
milk	1.00	1	3.8102	0.074672	.
sugar:milk	5.94	2	11.2769	0.001754	**
Residuals	3.16	12			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

10 Bioequivalence (BE) data example

(22) MODEL

```
GLM(log(CMAX) ~ SEQ/SUBJ + PRD + TRT, BEdata) # a BE dataset in sasLM package
```

\$ANOVA

Response : log(CMAX)

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
MODEL	48	23.1924	0.48317	5.6278	4.395e-08 ***
RESIDUALS	42	3.6059	0.08585		
CORRECTED TOTAL	90	26.7983			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

\$Fitness

Root MSE	log(CMAX)	Mean Coef Var	R-square	Adj R-sq
0.2930098	6.071036	4.826355	0.8654428	0.7116631

\$`Type I`

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
SEQ	1	0.6454	0.64544	7.5178	0.008938 **
SEQ:SUBJ	45	22.4395	0.49866	5.8081	3.359e-08 ***
PRD	1	0.0969	0.09686	1.1281	0.294242
TRT	1	0.0106	0.01057	0.1231	0.727410

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

\$`Type II`

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
SEQ	1	0.6440	0.64395	7.5005	0.009011 **
SEQ:SUBJ	45	22.5232	0.50052	5.8298	3.173e-08 ***
PRD	1	0.0996	0.09958	1.1599	0.287632
TRT	1	0.0106	0.01057	0.1231	0.727410

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

\$`Type III`

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
SEQ	1	0.3368	0.33679	3.9228	0.05421 .
SEQ:SUBJ	45	22.5232	0.50052	5.8298	3.173e-08 ***
PRD	1	0.0996	0.09958	1.1599	0.28763
TRT	1	0.0106	0.01057	0.1231	0.72741

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
options(contrasts=c("contr.sum", "contr.poly"))
```

```
Anova(lm(log(CMAX) ~ SEQ/SUBJ + PRD + TRT, BEdata), type=3, singular.ok=TRUE)
```

Note: model has aliased coefficients
 sums of squares computed by model comparison

Anova Table (Type III tests)

Response: log(CMAX)

	Sum Sq	Df	F values	Pr(>F)
SEQ	0.0000	0		
PRD	0.0996	1	1.1599	0.2876
TRT	0.0106	1	0.1231	0.7274
SEQ:SUBJ	22.5232	45	5.8298	3.173e-08 ***
Residuals	3.6059	42		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

11 Sesssion Information

R version 4.5.2 (2025-10-31 ucrt)

Platform: x86_64-w64-mingw32/x64

Running under: Windows 11 x64 (build 26100)

Matrix products: default

LAPACK version 3.12.1

locale:

[1] LC_COLLATE=Korean_Korea.utf8 LC_CTYPE=Korean_Korea.utf8

[3] LC_MONETARY=Korean_Korea.utf8 LC_NUMERIC=C

[5] LC_TIME=Korean_Korea.utf8

time zone: Asia/Seoul

tzcode source: internal

attached base packages:

[1] stats graphics grDevices utils datasets methods base

other attached packages:

[1] car_3.1-5 carData_3.0-6 sasLM_0.10.8 mvtnorm_1.3-3 rmarkdown_2.30

loaded via a namespace (and not attached):

[1] digest_0.6.39 Formula_1.2-5 fastmap_1.2.0 xfun_0.56

[5] abind_1.4-8 knitr_1.51 htmltools_0.5.9 cli_3.6.5

[9] compiler_4.5.2 tools_4.5.2 evaluate_1.0.5 yaml_2.3.12

[13] rlang_1.1.7 MASS_7.3-65